13-1 Experimental and Theoretical Probability

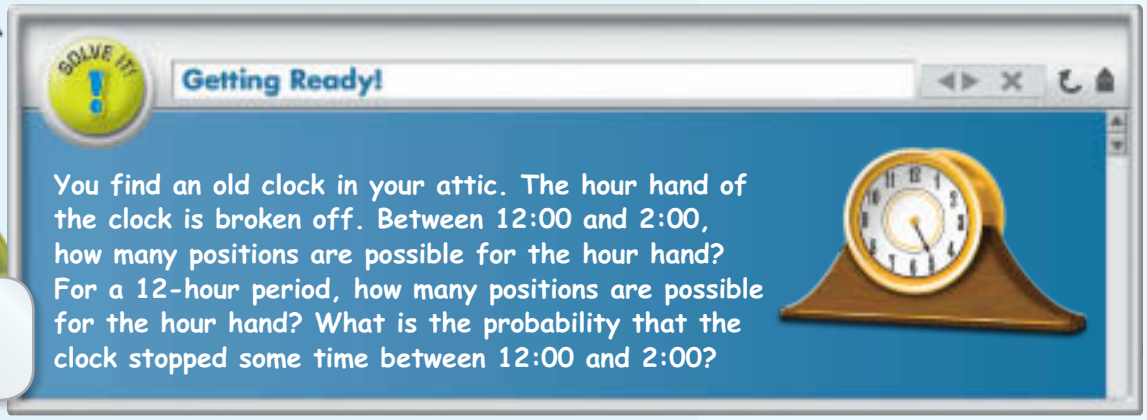
Common Core State Standards

S-CP.A.1 Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events. **Also S-CP.A.4**

MP 1, MP 2, MP 3, MP 4, MP 6

Objectives To calculate experimental and theoretical probability

Visualize where the hour hand could be.



MATHEMATICAL PRACTICES

In the Solve It, you probably considered where the hour hand would be based on where the minute hand is. In the language of probability, this position would be a *favorable outcome*. An **outcome** is the possible result of a situation or experiment. An **event** may be a single outcome or a group of outcomes. For the clock, the hour hand being about halfway between 12 and 1 or about halfway between 1 and 2 are two favorable outcomes for the event of where the hour hand may stop between 12:00 and 2:00. The set of all possible outcomes is the **sample space**.

Lesson Vocabulary

- outcome
- event
- sample space
- probability
- experimental probability
- theoretical probability
- complement of an event

Essential Understanding Probability is a measure of the likelihood that an event will occur.

take note

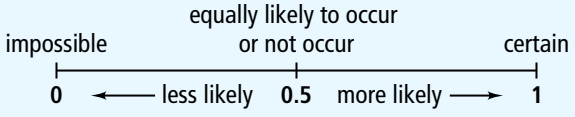
Key Concept Probability

Definition

If the outcomes in a sample space are equally likely to occur, the **probability** of an event $P(\text{event})$ is a numerical value from 0 to 1 that measures the likelihood of an event.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

You can write the probability of an event as a ratio, decimal, or percent.



You can find probabilities by using the results of an experiment or by reasoning mathematically.

take note

Key Concept Experimental Probability

Experimental probability of an event measures the likelihood that the event occurs based on the actual results of an experiment.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of times the experiment is done}}$$

Problem 1 Calculating Experimental Probability

Quality Control **A quality control inspector samples 500 LCD monitors and finds defects in three of them.**

A **What is the experimental probability that a monitor selected at random will have a defect?**

$$P(\text{defect}) = \frac{\text{number of monitors with a defect}}{\text{number of monitors inspected}}$$
$$= \frac{3}{500}$$
$$= 0.006 \text{ or } 0.6\%$$

Think

You can report the result as a fraction, decimal, or percent. Use the form that will communicate most clearly.

The experimental probability that a monitor selected at random is defective is 0.6%.

B **If the company manufactures 15,240 monitors in a month, how many are likely to have a defect based on the quality inspector's results?**

$$\text{number of defective monitors} = P(\text{defect}) \cdot \text{total number of monitors}$$
$$= 0.006 \cdot 15{,}240$$
$$= 91.44$$

It is likely that approximately 91 monitors are defective.

Got It? **1.** A park has 538 trees. You choose 40 at random and determine that 25 are maple trees. What is the experimental probability that a tree chosen at random is a maple tree? About how many trees in the park are likely to be maple trees?

When a sample space consists of real data, you can find the experimental probability. **Theoretical probability** describes the likelihood of an event based on mathematical reasoning.

This chapter uses *standard number cubes* to illustrate probability. A standard number cube has 6 faces with a number from 1 to 6 on each face. No number is used twice.

Problem 2 Calculating Theoretical Probability

What is the probability of rolling numbers that add to 7 when rolling two standard number cubes?

Think

What results should you be looking for?
Any two cubes that result in the sum of 7, like a 1 and a 6.

Step 1 Make a table of the possible results for the rolls of two number cubes. Circle the ones that sum to 7.

	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	(6,1)
2	1,2	2,2	3,2	4,2	(5,2)	6,2
3	1,3	2,3	3,3	(4,3)	5,3	6,3
4	1,4	2,4	(3,4)	4,4	5,4	6,4
5	1,5	(2,5)	3,5	4,5	5,5	6,5
6	(1,6)	2,6	3,6	4,6	5,6	6,6

Step 2 Find the number of possible outcomes for the event that the sum of two cubes is 7.

Step 3 Find the probability.

$$P(\text{rolling a sum of 7}) = \frac{6}{36}$$

The probability of rolling numbers that add to 7 is $\frac{6}{36}$, or $\frac{1}{6}$.

Got It? **2.** What is the probability of getting each sum when rolling two standard number cubes?

a. 9 **b.** 2 **c.** 13

The **complement of an event** consists of all of the possible outcomes in the sample space that are not part of the event. For example, if you roll a standard number cube, the probability of rolling a number less than 3 is $P(\text{rolling} < 3) = \frac{2}{6}$, or $\frac{1}{3}$. The probability of *not* rolling a number less than 3 is $P(\text{not} < 3) = \frac{4}{6}$, or $\frac{2}{3}$.

Take note

Key Concept Probability of a Complement

The sum of the probability of an event and the probability of its complement is 1.

$P(\text{event}) + P(\text{not event}) = 1$ $\qquad$ $P(\text{not event}) = 1 - P(\text{event})$

Problem 3 Using Probabilities of Events and Their Complements

A jar contains 10 red marbles, 8 green marbles, 5 blue marbles, and 6 white marbles. What is the probability that a randomly selected marble is not green?

Think

Can you find *P*(not green) another way?
You can find the total number of marbles that are not green, and then divide by the total number of marbles: $\frac{(10 + 5 + 6)}{29}$.

$P(\text{not green}) = 1 - P(\text{green})$ Probability of the complement

$= 1 - \frac{8}{29}$ Find *P*(green).

$= \frac{21}{29}$ Simplify.

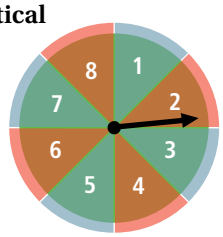
The probability that the chosen marble is not green is $\frac{21}{29}$.

Got It? **3.** What is the probability that a randomly chosen marble is not red?

Lesson Check

Do you know HOW?

Use the spinner to find each theoretical probability.



1. P(an even number)
2. P(a number greater than 5)
3. P(a prime number)

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** How are experimental and theoretical probability similar? How are they different?
5. **Open-Ended** Give an example of an impossible event.
6. **Error Analysis** Your friend says that the probability of rolling a number less than 7 on a standard number cube is 100. Explain your friend's error and find the correct probability.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

7. A baseball player got a hit 19 times of his last 64 times at bat.
   a. What is the experimental probability that the player got a hit?
   b. If the player comes up to bat 200 times in a season, about how many hits is he likely to get?

See Problem 1.

8. A medical study tests a new cough medicine on 4250 people. It is effective for 3982 people. What is the experimental probability that the medicine is effective? For a group of 9000 people, predict the approximate number of people for whom the medicine will be effective.

A bag contains letter tiles that spell the name of the state MISSISSIPPI. Find the theoretical probability of drawing one tile at random for each of the following.

See Problems 2 and 3.

9. P(M)
10. P(I)
11. P(S)
12. P(P)
13. P(not M)
14. P(not I)
15. P(not S)
16. P(not P)

B Apply

17. **Think About a Plan** Suppose that you flip 3 coins. What is the theoretical probability of getting at least 2 heads?
   - What is the sample space of possible outcomes?
   - What are the favorable outcomes?

18. **Music** A music collection includes 10 rock CDs, 8 country CDs, 5 classical CDs, and 7 hip hop CDs.
   a. What is the probability that a CD randomly selected from the collection is a classical CD?
   b. What is the probability that a CD randomly selected from the collection is not a classical CD?

19. You are playing a board game with a standard number cube. It is your last turn and if you roll a number greater than 2, you will win the game. What is the probability that you will not win the game?

STEM **20. Weather** If there is a 70% chance of snow this weekend, what is the probability that it will not snow?

21. Quality Control From 15,000 graphing calculators produced by a manufacturer, an inspector selects a random sample of 450 calculators and finds 4 defective calculators. Estimate the total number of defective calculators out of the 15,000.

22. Suppose you choose a letter at random from the word shown below. What is the probability that you will not choose a B?

PROBABILITY

A student randomly selected 65 vehicles in the student parking lot and noted the color of each. She found that 9 were black, 10 were blue, 13 were brown, 7 were green, 12 were red, and 14 were a variety of other colors. What is each experimental probability?

23. P(red)

24. P(black)

25. P(not blue)

26. P(not green)

STEM **27. Genetics** Genetics was first studied by Gregor Mendel, who experimented with pea plants. He crossed pea plants having yellow, round seeds with pea plants having green, wrinkled seeds. The following are the probabilities for each type of new seed.

yellow, round: 56.25% yellow, wrinkled: 18.75%

green, round: 18.75% green, wrinkled: 6.25%

If 2014 seeds were produced, how many of each variety would you expect?

Reasoning For Exercises 28–31, describe each of the following situations using one of the following probabilities. Explain your answer.

I. 0 **II.** between 0 and 0.5 **III.** between 0.5 and 1 **IV.** 1

28. having school on Tuesday

29. two elephants in the city zoo having the same weight

30. getting your driver's license at the age of 10

31. turning on the TV while a commercial is playing

Challenge

32. The students in a math class took turns rolling a standard number cube. The results are shown in the table at the right.

a. What is the theoretical probability of rolling the number 1 with the number cube?

b. What was the experimental probability of rolling the number 1 for the experiment in class?

Number Cube Experiment

Outcome	1	2	3	4	5	6
Times Rolled	39	40	47	42	38	44

33. Another way to express probability is with *odds*. Odds compare the number of favorable outcomes to the number of unfavorable outcomes. Odds in favor of an event are usually written as

number of favorable outcomes : number of unfavorable outcomes.

Suppose the probability of drawing a red marble from a bag of marbles is $\frac{3}{10}$.

a. What are the odds in favor of drawing a red marble?

b. What are the odds against drawing a red marble?

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 1, MP 4

Look back at the information on page 823 about the survey Lorreta's Ice Cream conducted. Copy and complete the table below.

Flavors Liked	Number of Responses
Only Raspberry	■
Only Caramel	■
Both	■
Neither	■

Select all of the following that are true. Explain your reasoning.

A. An equation that can be used to find the number of customers in the survey that like both flavors is $72 + 3x = 300$, where x is the number who like both.

B. The number of customers in the survey that like both flavors is 57.

C. The number of customers in the survey that like both flavors is 76.

D. The experimental probability that a customer who tries both new flavors will like only raspberry is $\frac{6}{25}$.

E. The experimental probability that a customer who tries both new flavors will like only raspberry is 0.72.

F. The experimental probability that a customer who tries both new flavors will like only caramel is 57%.

G. The experimental probability that a customer who tries both new flavors will like only caramel is 0.38.